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Life Symptoms: the Behaviour of Open Systems with Limited Energy Dissipation Capacity and Evolution*

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Abstract — For income distributions, city-size distributions and word frequencies we observe skewed or long-tailed distributions of the PARETO-ZIPF type. In general – for properties of units belonging to the same astrophysical, biological, ecological, urban, social, political, economic or mechanical machinery system – we observe similar regularities referred to as Pareto law, Zipf's law or rank-size law.
We give a short historical overview over the discovery of these empirical regularities. Then we review a variety of theoretical tentatives, none of which can "explain" that similar regularities are observed in such an incredible wide range of scientific research areas.
Finally, departing from a very specific model – describing tribo-vibro-acoustic processes in machines – we propose a generalized theoretical framework in terms of energy transformation with limited internal energy dissipation capacity, which is applicable to all fields.
The proposed model "unifies" a large variety of concepts and applies a coherent terminology to fields, which have at first sight nothing in common. For the observed life symptoms, theoretical predictions can be compared with past and future empirical observations.
What is most important is the model's inference power: from the observations of a set of units at a given moment of life-time (a snapshot of the system), one can predict the average behaviour of a single unit over its entire life-time.

Keywords — Self-organization, evolution, energy transformation, energy dissipation, internal structure, information, autocatalysis, Pareto-Zipf or rank-size distributions, machine vibration diagnostics, birth and death processes, symptom-life curve, residual life-time.
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1 Introduction

What do vibrations of electro-motors and machines in general have in common with atoms of stars, with chemical compounds of the ocean, with animals of biological

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species, with inhabitants of cities, with market values of firms, with words of a language ...?
A reductionist scientist would consider the above question as ridiculous, if not insane.
And a "systems scientist"?

Any scientific activity – including systems science – is driven by:

i) a strive for **economy of thought** (from specific observations to general models or theories): any model or theory "compressing" the description and "simplifying" the comprehension of present or past observations can be considered as scientific
ii) a strive for **predictability** (from general models or theories to specific observations): any model or theory must be able to be confronted to specific past or future observations.

The objectives of systems science are still the same as stated almost 40 years ago by the *Society for General Systems Research*:

1 to investigate the **isomorphy of concepts**, laws and models from various fields, and to help in useful transfers from one field to another
2 to encourage development of **theoretical models** in fields which lack them
3 to **minimize duplication** of theoretical efforts in different fields, and
4 to promote the **unity of science** through improving communications among specialists.

If we want transdisciplinary questions to be taken seriously by the "traditional" compartments of science, we must be able to produce systems theories, which are more than simplified diagrams of feedback loops, verbal streams of systems jargon or terribly complicated sets of differential equations without any reference to experimental data.
To put it in a more "fashionable" way, *scientific* system models or theories must be *falsifiable* in the sense of Popper. This crucial aspect of science is unfortunately quite often neglected, not only in "systems" literature.

2 Empirical observations of Pareto-Zipf distributions

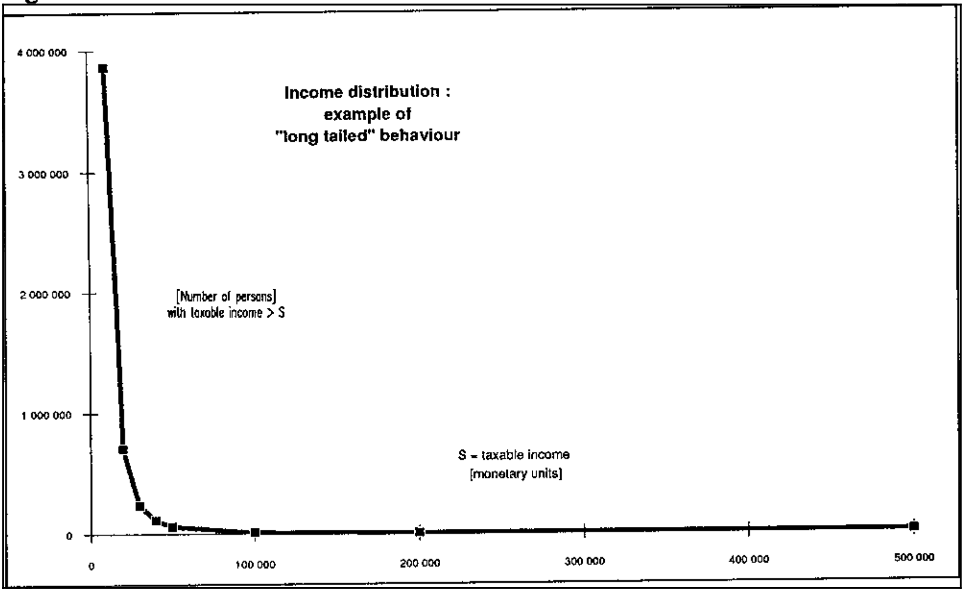
2.1 *Pareto, the distribution of incomes in economic systems*
The first extensive discussion of the problem how income is distributed among the citizens of a state was made by Vilfredo Pareto[9] in 1897. On the basis of data collected from numerous sources Pareto arrived at the following law:

In all places and at all times the distribution of income in a stable economy, when the origin of measurement is at a sufficiently high income level, will be given approximately by the empirical formula

(1) $\mathbf{n} = \mathbf{a}\,\mathbf{S}^{-\gamma}$

where **n** is the number of people having the income **S** or greater, **a** and γ are constants.

Figure 1 *Income distribution is an example of a skewed or "long-tailed" distribution.*



Note, that it is difficult to represent the data graphically within ordinary arithmetic scales. The data are taxable incomes of 1937 in France, but any other country and year yields distributions of the above type

It is extremely interesting to note, that empirical observations of Pareto distributions are:

i) not markedly influenced by the socio-economic structure of the community under study

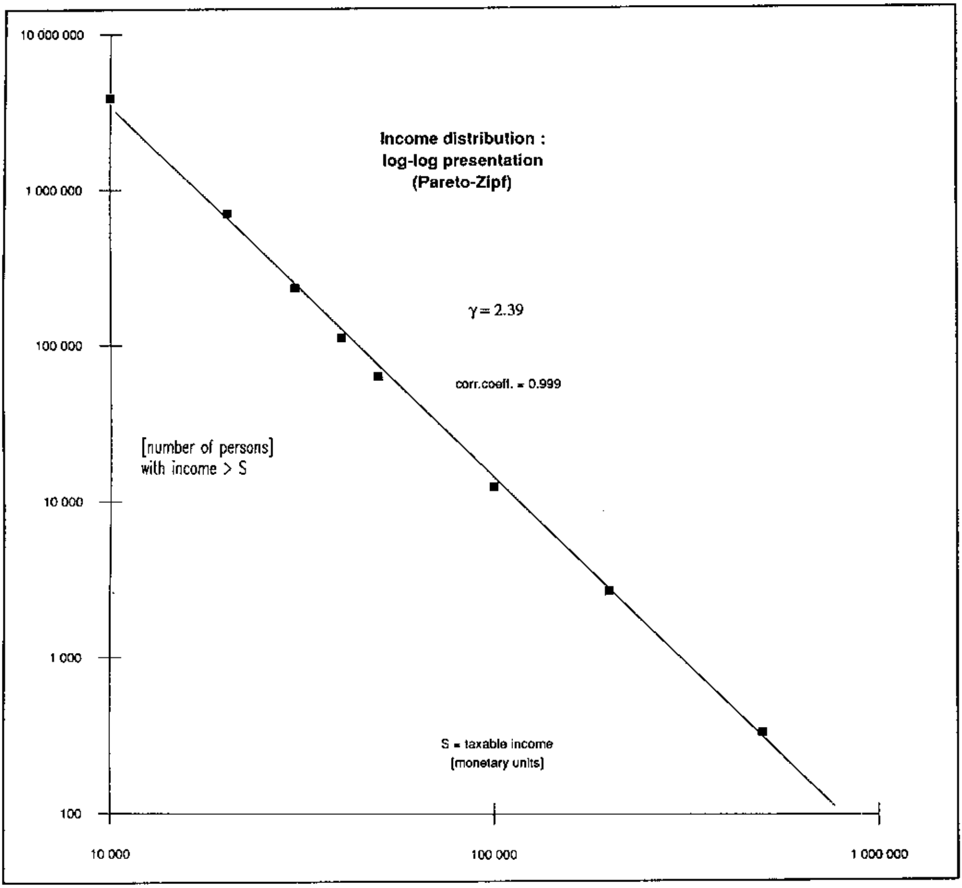
ii) not markedly influenced by the definition of "income".

The Pareto law holds for a few hundred burghers of a city-state of the Renaissance up to the more than 100 million taxpayers in the USA. Essentially the same law continues to be followed by the distribution of "income", despite the changes in the definition of this term.

Note: this empirical evidence is a contradiction to any ideology striving for equal distribution of incomes. As we shall see below, this goal is just as unrealistic and *unnatural* as the goal to make all cities of a country of the same number of inhabitants, to make all business firms of equal size or to use in a text all words with equal frequency.

Pareto was intrigued by the generality of his discovery: "These results are very remarkable. It is absolutely impossible to admit that they are due only to chance. There is most certainly a *cause*, which produces the tendency of incomes to arrange themselves according to a certain curve."

Figure 2 *The data of Figure 1 in a log-log presentation. γ is the exponent of the Pareto-Zipf law. Note the almost "perfect" correlation coefficient*



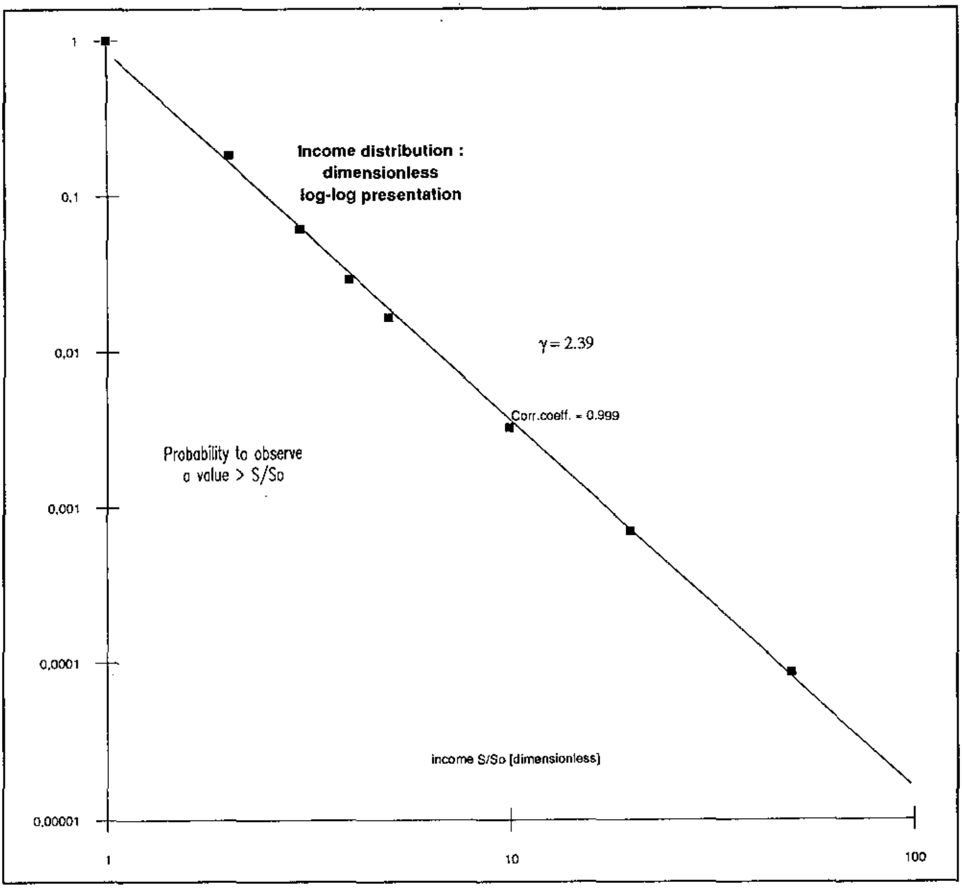
1.2 *Auerbach, the distribution of city sizes in countries*

Looking for a new measure for population concentration, Auerbach[1] analyzed the distribution of cities within a country. He ranked the cities in decreasing order of inhabitants and discovered a relationship between rank and size of the type

(2) $\mathbf{S(j) = a\, j^{-\beta}}$

with **S(j)** the size of the city ranked **j**.
a and **β** are constants.

Figure 3 *shows the same data as in Fig. 2 after normalization. The graph shows the cumulative probability as a function of the dimensionless symptom S/So. S is the income and So the lowest income of the observed set. For transdisciplinary comparisons we prefer the normalized presentation*



As an example let us consider the city-size distribution of France (see also Figure 4.):

Table 1 *Example of a rank-size distribution*

City-size distribution	Rank j	Size S
Paris	1	8 549 898
Lyon	2	1 170 660
Marseille	3	1 070 912
Bordeaux	4	935 882
Toulouse	5	612 456
...	...	...

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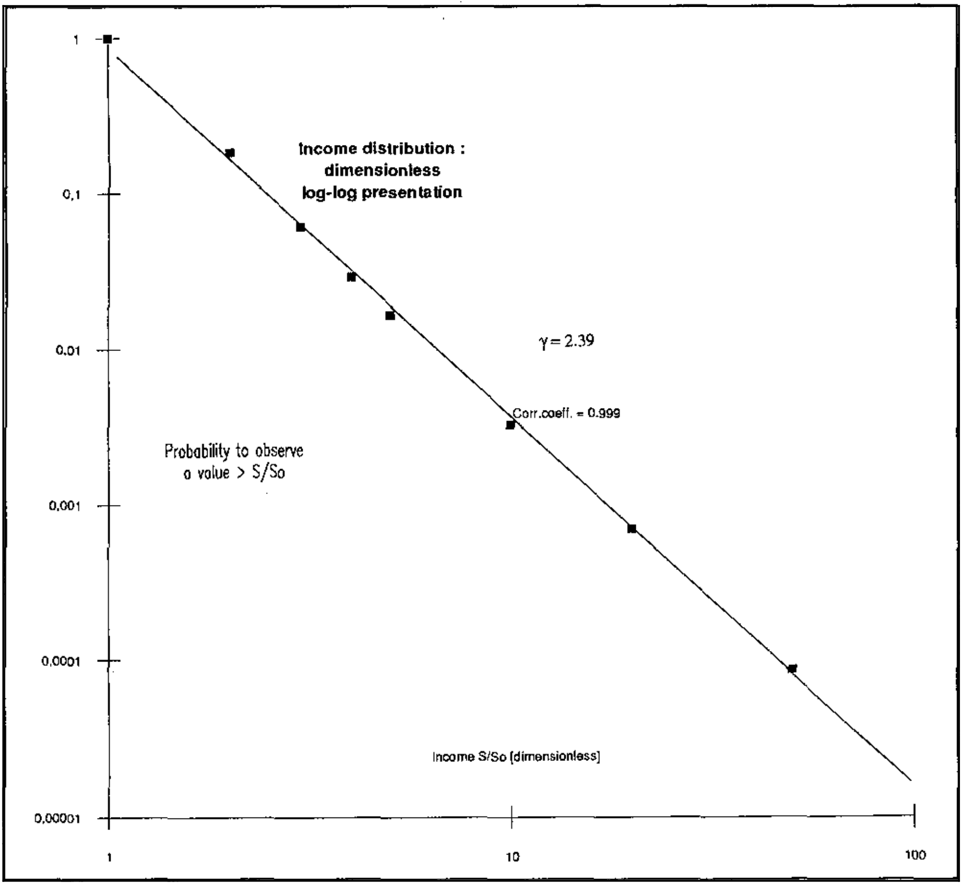

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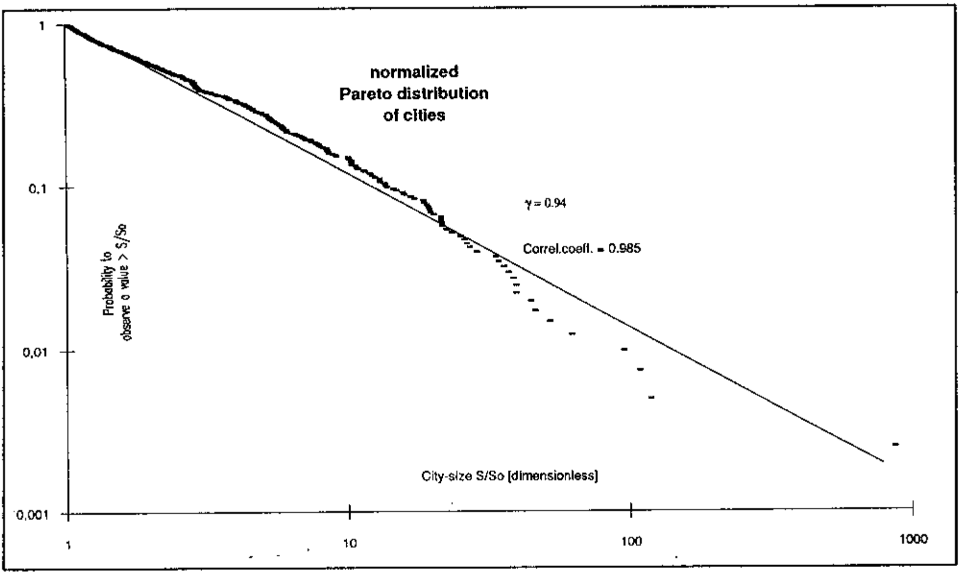
Figure 5 *Data of Figure 4 transformed to the normalized domain of probability. S is the city-size and S_o the smallest or threshold size for cities (here 10 000)*



1.3 *Willis-Yule, the distribution of species, genera and families in biological systems*

Based on field observation in Ceylon in 1912 Willis[15] first noticed, that the distribution of species within the genera of an ecosystem follows a regularity, which is of the Pareto-Zipf type.

" ... this type of curve holds not only for **all the genera of the world**, but also for all the individual families both of plants and animals, for endemic and non-endemic genera, for local floras and faunas ... it obtains too, for all the deposits of Tertiary fossils examined."

Further analysis of data have shown[20,21], that similar regularities hold also for the distribution of parasites on hosts, the distribution of individuals within species and the distribution of genera within families of **any observed ecosystem at any time.**

1.4 *Zipf, the distribution of words in languages*

In his magnum opus Zipf[23] reports regularities of the above type for a wide variety of fields, but his main interest is human language for which he analyzed word-frequency distributions.

James Joyce's *Ulysses* is the "richest" known text with almost 30 000 words and word occurrencies ranging from 1 to 2 653. The empirical data can be approximated almost too perfectly by a Pareto-Zipf distribution.

Zipf found regularities of similar type for all types of English text, for **all types of languages and for all times,** even for Chinese text and also for spoken language of children of different ages. The exponent γ is in all cases close to 1.

The only exceptions reported by Zipf are texts written by schizophrenics and scientific English!

Figure 6 *Species-size distribution of Macrolepidoptera,data from[20]., transformed to cumulative probability domain. 15 609 individuals were captured belonging to 240 species*

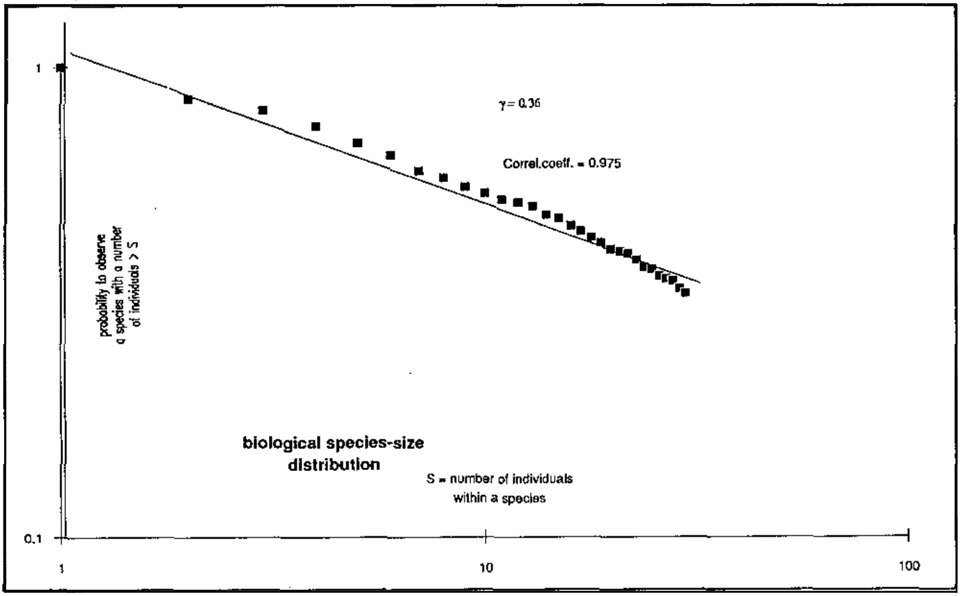
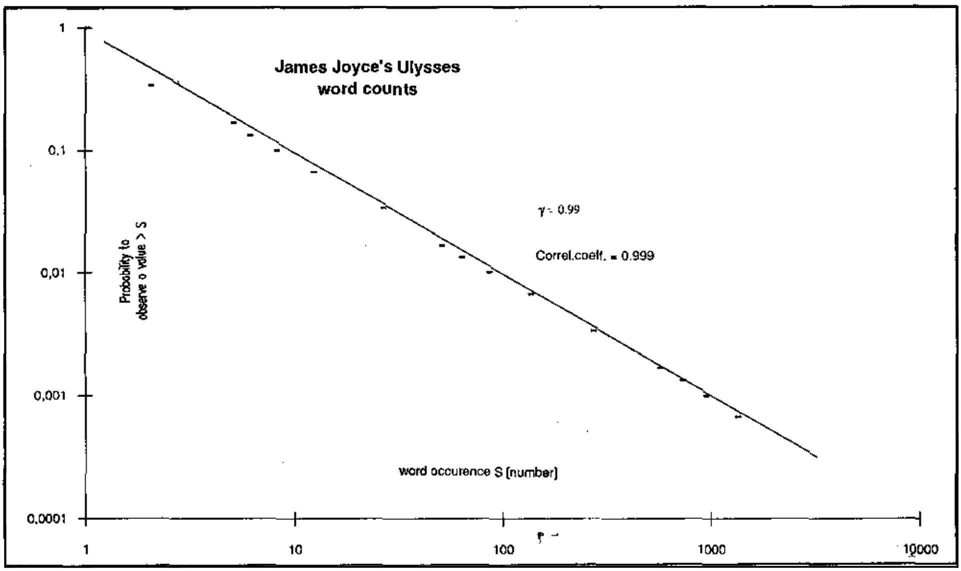


Figure 7 *Word counts for texts in any language yield Pareto-Zipf distributions. In normalized form the graph shows the probability of a word to occur more than S times in the text.*



In this particular case S = S/So, since the lowest occurrence of a word So = 1. For example the word Quarks occurs only once in the entire text Data from Zipf[23], transformed to cumulative probability domain

Zipf also reports, that the distribution of scientists within a research discipline is of Pareto-Zipf type. The observed "symptom" of a scientist is measured as the number of citations in the physical or chemical abstracts.

Writing this paper, the first author has discovered that the size-distribution of programs on the hard disk of a computer are of Pareto-Zipf type. Program-size is measured in kilobytes. For PC users one has to combine .EXE and .COM files in order to observe an almost perfect regularity.

1.5 *Simon, the size distribution of business firms*
Herbert Simon, who won the nobel prize for economics in 1978, has intensively studied firm-sizes:
Whether sales, assets, number of employees, value added, profits, or capitalization are used as a size measure, the observed distribution always are of the Pareto-Zipf type. This is true for the data for **individual industries** (economic sectors) and for **all industries taken together**. It holds for sizes of plants as well as of firms[13]. Take any annual number of the Fortune 500 magazine and you can verify this assertion, which also holds for any national economy and also for multinational companies on a world level.

We have analyzed the Fortune data over a period of 30 years[17] and found, that the parameter γ of the size-distributions remains almost constant. This self-similarity of the distribution curves holds in periods of overall economic growth as well as in periods of economic recession and despite the fact, that firms appear and disappear. From the 50 largest industrial firms in 1954 only 20 can be found among the 50 largest 3 decades later, the other 30 have declined in size, been absorbed in mergers and acquisitions or simply have gone out of business. On the other hand, 12 of the 50 largest firms were not even ranked among the 500 largest in 1954 or did not even exist at that time.

To observe a constant size-distribution despite this intensive shuffling around within the system is quite remarkable.

As Herbert Simon stated in the conclusion of his paper: "We need to know more about the relations between the distributions and the **generating processes**".

Since the graphs of the empirical data are monotonously similar, we will not burden the reader with examples.

Over time, the Pareto-Zipf line seems to act as an **attractor** for "deviating points"[17]. For example in the computer industry we had a similar situation as in the case of the largest French cities in Figure 4. IBM was "too big" and the next ten following companies were "too small". The evolution of the last 10 years has brought the "deviations" almost *back in line* again due to:
i) a relative decline of the growth rate of IBM
ii) an above average growth rate of DEC
iii) several mergers and acquisitions among the top computer companies.

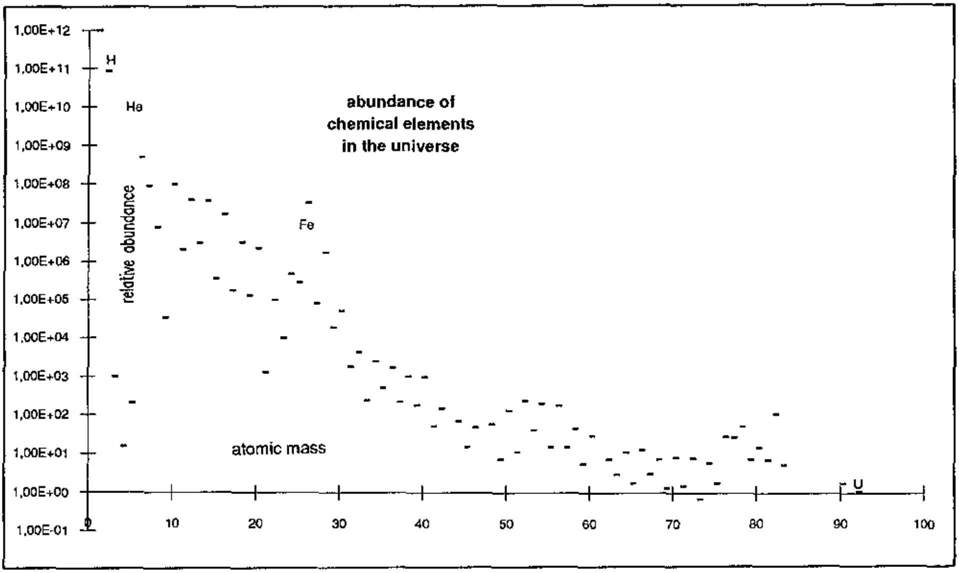
1.6 *Gutenberg-Richter, the distribution of earthquakes*

In 1956, the geologists Beno Gutenberg and Charles Richter (the father of the seismological scale of the same name) discovered[6], that the number of important earthquakes is linked to the number of small earthquakes:
the *law of Gutenberg-Richter* states, that the number of annual earthquakes as a function of the liberated Energy, is a Pareto-Zipf-distribution. The exponent $\gamma = 1.5$ is universal and **does not depend on the geographical region!**

1.7 *Winiwarter, the distribution of elements in cosmic systems*

The analysis of chemical element distributions within stars or within the entire cosmos is traditionally presented as relative abundance versus the mass number of the elements.

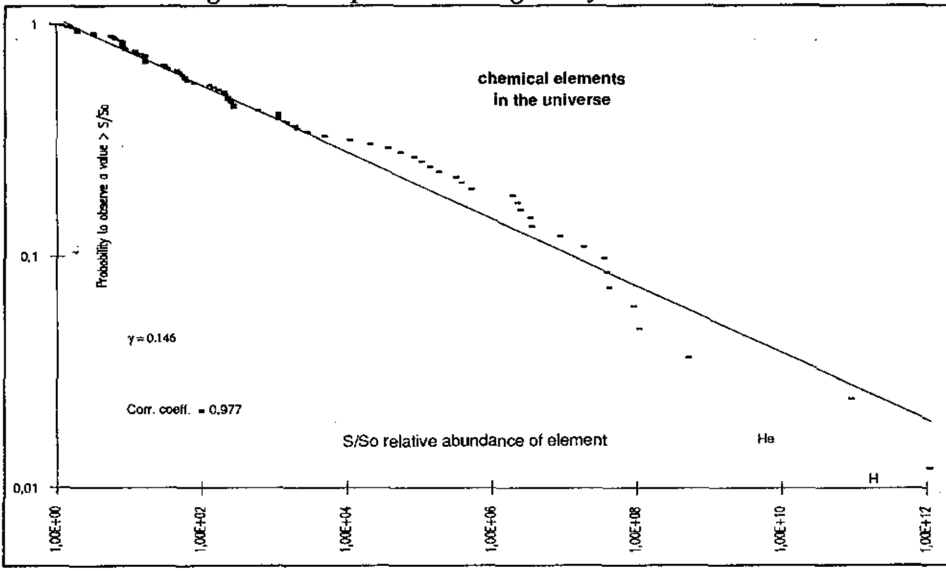
Figure 8 *Distribution of chemical elements in the universe*



This traditional representation of the data - showing relative abundance as a function of atomic mass - does not allow to deduce any quantitative regularity. data compiled by J.P.Meyer and A.G.W. Cameron[20]

From the above type of presentation one can deduce only a qualitative statement, that the abundance of elements has a tendency to decrease strongly with mass-number, with exceptional peaks around the *magic numbers.*
This type of regularity for the abundance of chemical elements can be observed for the universe, for single stars, for meteorites, for the lithosphere ...
Similar regularities can be observed for star-size distributions in galaxies, for the planet-size distribution in our solar system, for the lunar-size distributions of the Jupiter system ...

Figure 9 *The same data as in Figure 8 as a normalized Pareto-Zipf distribution revealing a distinct quantitative regularity*



1.8 *Cempel, the distribution of vibration amplitudes in mechanical machine systems*
Research in the field of vibration diagnostics[4] has revealed, that long-tailed Pareto-like distributions are a good approximation for the data yielded by empirical measurements of vibration symptoms for a set of "running" machines:
the regularities are observed independently of the machine type (electro motors, diesel engines ...)

1.9 *Do we live in a Pareto-Zipf world?*
The observation of similar phenomena for incomes, cities, species, words, earthquakes, chemical elements, machine vibrations: how can this possibly make sense?

3 Models "explaining" Pareto-Zipf distributions

3.1 *Mathematical form of the distributions: limits of empirical curve-fitting*
A variety of mathematical distributions has been proposed to fit empirical long tailed distributions of the discussed type. Quandt[11] compared more than 8 different mathematical functions and comes to the conclusion, that all the proposed laws are very similar. If one law yields a good adjustment of the data, then this is also the case for several other laws.

Searching for "best" empirical fits does not lead to a better comprehension of the observed phenomena. Unless there is strong theoretical evidence, the most simple "law" like the Pareto-Zipf distribution is to be preferred to more complicated

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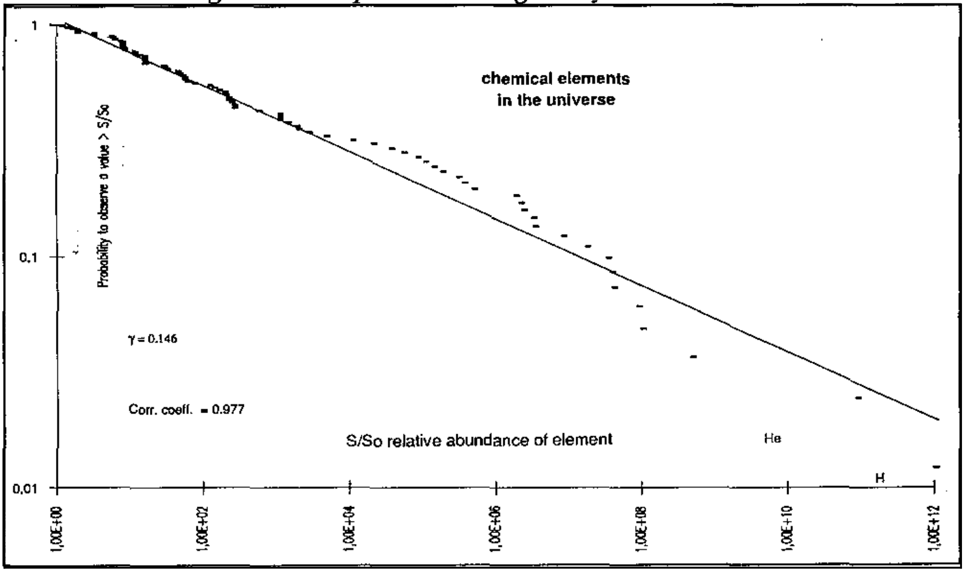

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3.4 *Mandelbrot-Winiwarter: first attempt, a holistic extremum principle*

Mandelbrot, the inventor of Fractals[7], has studied in detail the theory of coding and given an explanation for the regularities of word counts in terms of an extremum principle:

within a text, the quantity to be optimized / minimized is the "average cost per word".

Assuming that the "cost" of a word depends on the "costs" of its constituting letters, Mandelbrot showed that the resulting "optimal" distribution is of the Pareto-Zipf type.

Based on a general principal of evolution or self-organization which states, that the complexity of a self-organized system can only grow or remain constant (*first law of genesis*)[19], we put forward the hypothesis, that **Pareto-Zipf type distributions are common to all processes of self-organization** (*second law of genesis*)[20].

Generalizing Mandelbrot's arguments from words to energy quanta, we speculated, that the observed Pareto-Zipf regularities are the result of a general extremum principle, which maximizes what we have called the *energy redundancy* (binding energy or synergy) within a self-organized system.

This approach seems very general and attractive, however - besides for systems of nucleons - it is difficult or impossible to verify.

3.5 *Roehner-Winiwarter: second attempt, Pareto + Pareto = Pareto*

The Gaussian distribution is known to be a limit distribution of random variables. It is well known, that the random sum ⊕ of two Gaussian distributions G_1 and G_2 yields a new distribution G_3 which is also Gaussian.

$G_1 \oplus G_2 = G_3$

It is too generally assumed, that this property is unique for the distributions called "normal", "bell-shaped" or Gaussian.

We have shown, that Pareto distributions are possible limit distributions of sums of random variables[16].

The random sum ⊕ of two Paretian distributions P_1 and P_2 yields a new distribution P_3 which is also Paretian.

$P_1 \oplus P_2 = P_3$

Based on this statistical **stability** of Pareto distributions, we have explained the stability of empirical distributions as the result of a stochastic process:

$S_{t+1} = \alpha\, S_t \oplus \Delta$

The distribution at time t+1 depends on the distribution at time t multiplied by a factor a characterizing the total growth of the system, plus a deviation Δ added at random.

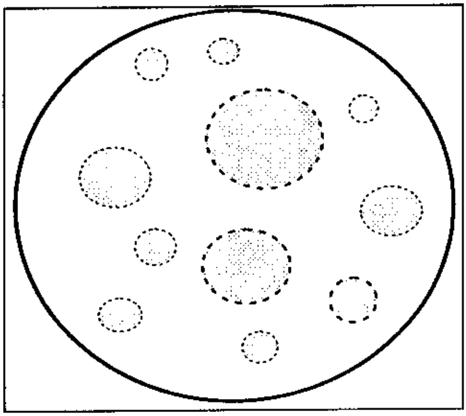
If the initial distribution is Paretian and if the distribution of fluctuations Δ is Paretian, then the resulting distribution must also be Paretian.

This statistical stability is certainly an interesting and important feature, explaining the extreme *perseverance* of Pareto- distributions over time, but it does not explain in a satisfactory way their origins.

Stating that every observed regularity is the stochastic result of prior regularities, can be mathematically correct, but is not a very satisfying explanation.

With respect to our first attempt however, it has become clear, that the following features are characteristic for Pareto-Zipf regularities in all observed fields:

Figure 10 *Three level description of a hierarchical system: elements / quanta (small dots, e.g. inhabitants), units (dotted circles, e.g. cities), system (fat circle, e.g. country)*



i) 3 level hierarchical description (see Figure 10.):
   - *elements* or *quanta*
   - *units*
   - *system* with observable boundary
ii) birth and death of units as part of an irreversible evolutionary process
iii) holistic or global system behaviour (energy optimization).

3.6 *Cempel-Winiwarter: third attempt, open energy transformation systems with limited dissipation capacity*

In the following we present a model which has been originally developed to explain the observed symptoms of "running" machines of equal type.
We shall then generalize the assumptions of the machine model to any system of energy transformation units.

4 The TVA model: Life and death of machines

The wear of a machine, and thus its condition, is related to the energy dissipated in the *tribo-vibro-acoustical* processes taking place within the machine. *Tribo* means rubbing, *Vibro* means vibrating and *Acoustical* means noise (just think of the breakdown of your last car).

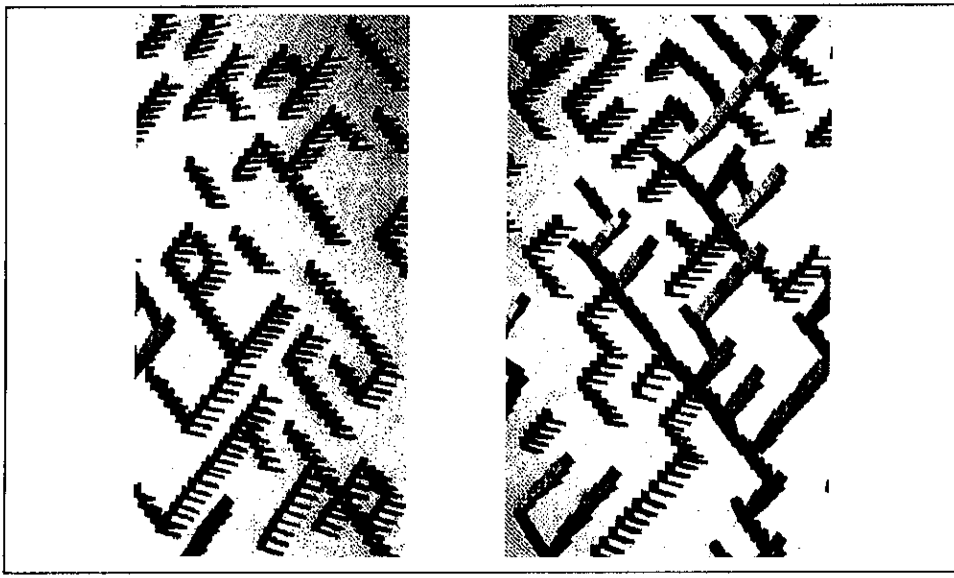
This approach was the basis for the elaboration of the *TVA* machine model in analytical form. The model allows to determine the condition of a machine from "passive" diagnostic experiments (i.e. measurements which are non-destructive and which do not interrupt the running of the machine) and it may be used for condition forecasting[3].

4.1 *A single unit "running to death": a machine as an open energy transformation system*

We describe a mechanical machine as an open system in terms of energy flows. The energy flows cover all types of energy such as kinetic energy and chemical energy.
Time is measured as *internal time* Θ of the system life. Energy flow per internal time unit is expressed in terms of *power* $\mathbf{N = dE / d\Theta}$.

For example: micro-fissions accumulate, forming bigger and bigger macro-fissions. When one of the macro-fissions covers at least half the diameter of a rotating axis, the system breaks down. (see Figure 14 below).

Figure 14 *Infra-Structures or In-Formation: micro-fissions as an example for the accumulation of internal structure: independent islands of micro-fissions (left) grow into a continuous macro-fission leading to the breakdown of the structure (right)*



The differential increment of internally accumulated dissipated energy is formally:

$$(2)\quad dE_{int}\left[\Theta,V(\Theta)\right] = \frac{\delta}{\delta\Theta}E_{int}\left[\Theta,V(\Theta)\right]d\Theta + \frac{\delta}{\delta V}E_{int}\left[\Theta,V(\Theta)\right]dV$$

Replacing on the right hand side E_{int} according to expression (1) we get

$$(3)\quad dE_{int}\left[\Theta,V(\Theta)\right] = D\left[\Theta,V(\Theta)\right]d\Theta + \frac{\delta}{\delta V}\left\{\int_0^\Theta D\left[\Theta,V(\Theta)\right]d\Theta\right\}dV$$

It is known from tribology, that the intensity of the internal wear process (its power D) is governed mainly by the power V of the externally dissipated output (vibration, heat etc.)

$$(4)\quad D\left[\Theta,V(\Theta)\right] = D\left[V(\Theta)\right] + \varepsilon D\left[\Theta,V(\Theta)\right] \quad \text{with } \varepsilon < 1$$

We therefore can assume in the first approach, that the transformation law of internally accumulated dissipated power D and externally dissipated power V is

constant during the lifetime Θ of the system: hence is not a function of Θ and can be replaced by .
This simplifies expression (3) end we get

$$(5)\quad dE_{int}[\Theta,V(\Theta)] = D[\Theta,V(\Theta)]d\Theta + \Theta\frac{d}{dV}D[V(\Theta)]dV$$

as expression for the differential behaviour of internally accumulated dissipated energy.

4.3 "Wear" amplifies "wear", positive feedback leading to autocatalytic or non-linear behaviour

Postulate 2: The externally dissipated power **V** is proportional to the amount of dissipated energy $\mathbf{E_{int}}$ accumulated internally:

$$(6)\quad \mathbf{dV(\Theta) = \alpha\, dE_{int}(\Theta)}$$

with α = constant at first approach.

This means, that the elementary increase of internally accumulated dissipated energy $\mathbf{dE_{int}(\Theta)}$ is related to the energy (or power) increase of external dissipation $\mathbf{dV(\Theta)}$, by the conversion coefficient **a.**
Replacing the differential $dE_{int}(\Theta)$ in (6) according to (5) we obtain

$$(7)\quad dV(\Theta) = \alpha\left\{D[\Theta,V(\Theta)]d\Theta + \Theta\frac{d}{dV}D[V(\Theta)]dV\right\}$$

and the differential equation for external dissipation power

$$(8)\quad \frac{dV(\Theta)}{d\Theta} = \frac{\alpha D[\Theta,V(\Theta)]}{1-\alpha\Theta\frac{d}{dV}D[V(\Theta)]}$$

4.4 Breakdown time

The denominator of (8) vanishes for

$$\alpha\Theta\frac{d}{dV}D[V(\Theta)] = 1$$

According to our assumptions α and are constant over internal time Θ and we can define the ***breakdown time*** Θ_b:

$$(9)\quad \Theta_b = \frac{1}{\alpha\Theta\frac{d}{dV}D[V(\Theta)]} = \frac{dE_{int}}{dD}$$

As it is seen, the break down time is determined by the internal structure of the system and the way of energy dissipation inside the system.

4.5 Differential equation governing dissipation

Introducing (9) into (8) we get

$$(10)\quad \frac{dV(\Theta)}{d\Theta} \quad \frac{\alpha D\left[\Theta, V(\Theta)\right]}{1-\Theta/\Theta_b}$$

Solving equation (10) with respect to internally accumulated dissipated power **D** we obtain

$$(11)\quad D(\Theta) = \frac{D_0}{1-\Theta/\Theta_b}$$

with $0 \leq \Theta < \Theta_b$ and D_0 the internally dissipated power D at time $\Theta = 0$.

With respect to external dissipation power V we obtain

$$(12)\quad V(\Theta) = \frac{V_0}{1-\Theta/\Theta_b}$$

with $0 \leq \Theta < \Theta_b$ and V_0 the externally dissipated power V at time $\Theta = 0$.
It is seen from above that both system variables D and V are colinear and reach infinity at the breakdown time $\Theta = \Theta_b$.

4.6 *The symptom life curve*

In practice it is impossible to observe directly the total dissipated powers $V(\Theta)$ or $D(\Theta)$ in particular. Instead, we are observing a symptom of the wear process like vibration, noise, temperature, voltage etc.
Let us assume, that the chosen symptom $S(\Theta)$ is a function of $V(\Theta)$, which can at the first approach be approximated by an exponent relation

$$(13)\quad S(\Theta) = \Phi[V(\Theta)] = [V(\Theta)]^{1/\gamma}$$
$$\text{with}\quad S_0 = \Phi(V_0)$$

Introducing (13) in (12) we obtain the *symptom life curve* of the system:

$$(14)\quad \frac{S(\Theta)}{S_0} = (1-\Theta/\Theta_b)^{-1/\gamma}$$

which is shown qualitatively in Figure 15.

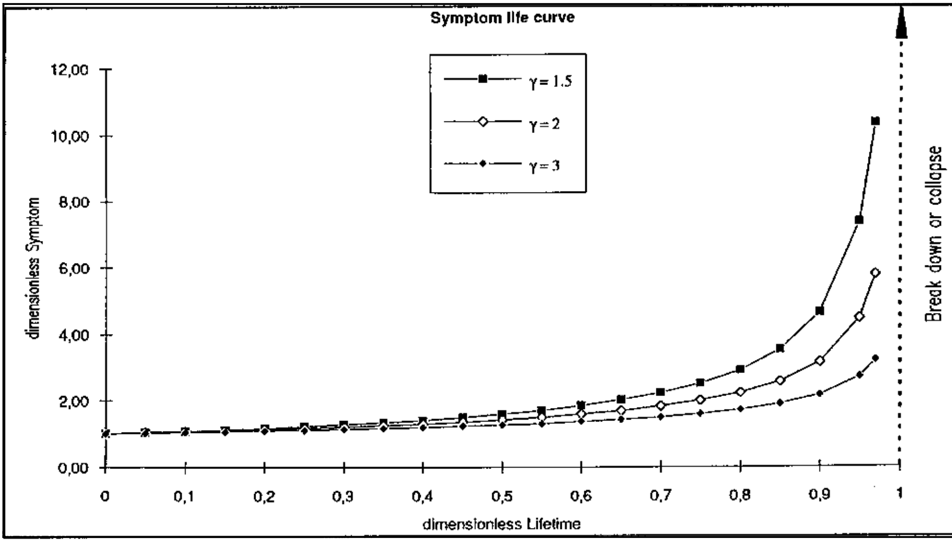
If one has the choice among different observable symptoms, one should select the one with the lowest γ value (most sensitive) for good diagnostic and prognostic results[4].

4.7 *A group of "running" units: the cumulative symptom probability curve*

In very rare cases we have the opportunity to observe the same object over its entire life-time.
Usually we observe a group of objects of the same type. So instead of observing a single deterministic process of symptom life development, we are observing an auto regressive stochastic process $S = S\ (\Theta/\Theta_b, \Delta)$.

Here Θ/Θ_b is the dimensionless lifetime, which monotonically increases, and Δ is a vector of components, which are specific for each individual unit.

Figure 15 *Life time behaviour of symptoms of "wear". Every type of symptom approaches infinite value asymptotically at the vicinity of the breakdown time $\Theta/\Theta_b = 1$. The less the value of the exponent γ, the more sensitive is the symptom*



In the case of machines, these components influencing the life of a machine, may be small differences in quality (a "Monday car"), differences of foundations (support), differences in load intensity and differences in the quality of maintenance for the individual units, to name only the most important.

For one unit, these components and many other factors, might be known and considered as deterministic. For a group of units, we have to acknowledge them as random, due to our lack of knowledge.

Taking a statistical approach instead of the deterministic one, we consider a group of N>>1 different units of the same type being each at different life-time stage. Measuring the empirical symptom value S_e for each unit, we can make an ordered empirical statistics of the type

$$(15) \quad P(S_e \geq S) = \int_S^{\infty} p(S_e)dS_e = \frac{n(S_e \geq S)}{N}$$

Where $P(S_e \geq S)$ is the cumulative probability to observe empirical symptom values S_e which are greater or equal to a prescribed value S, and n (.) is the number of machines with that property.

The cumulative probability P can be expressed as the probability density $p(S_e)$ integrated from S upward to infinity.

For a given set of discrete observations, we approximate this integral by the ratio

of units n(.) with an observed symptom value superior or equal to S, divided by the total number of units observed N.
Note: in engineering the probability (15) is called *reliability* ; it is simply an empirical estimate for the correct functioning of a machine for which the observed symptom S_e exceeds a value S.
The general statistical relationship[8]
(16) $p(S)\, dS = p(\Theta)\, d\Theta$
gives us the possibility to calculate the probability density in the time domain Θ and in the symptom domain S.
For the symptom domain we get

$$(17)\quad p(S) = p(\Theta) \left|\frac{d\Theta}{dS}\right|$$

where $p(\Theta)$ is the density of observation in the time domain. The derivative $\left|\frac{d\Theta}{dS}\right|$ can be calculated from the symptom live curve (14) as

$$(18)\quad \frac{d\Theta}{dS} = \gamma\frac{S_0}{S^2}\left(\frac{S_0}{S}\right)^{\gamma-1}$$

We assume the density of empirical observations to be of the form

$$(19)\quad p(\Theta) = \left(\frac{S(\Theta)}{S_0}\right) \quad ;\ \text{with } n \geq 0.$$

Putting now (19) and (18) into (17) and the result into the integral of (15) one can find for the cumulative symptom distribution

$$(20)\quad P(S_e \geq S) = \int_S^\infty \gamma\frac{S_0}{S^2}\left(\frac{S_0}{S}\right)^{\gamma-1-n} dS_e = \frac{\gamma}{\gamma-n}\left(\frac{S_0}{S}\right)^{\gamma-n} \quad ;\ \text{with } \gamma \text{ and } n \geq 0.$$

One can see from above, that independently of the way of observation - the exponent n in (19) - **the cumulative symptom probability distribution is of Pareto type.**
For the simplest condition, where the frequency of observations of a unit is not related to the symptom-life behaviour (n = 0), one can obtain the following behaviour for the *cumulative symptom distribution*

$$(21)\quad P(S_e \geq S) = \left(\frac{S_0}{S}\right)^{\gamma} = \left(\frac{S}{S_0}\right)^{-\gamma} \quad ;\ \text{with } \gamma > 0.$$

This means: **if the symptom-life curve of a unit is of the type (14)** - due to a limited potential of energy dissipation - **then the cumulative distribution of symptoms for a group of units will be of Pareto type.**

4.8 *Inference: predictive power*
According to our theory the exponent γ in (21) is the same as the exponent γ in (14) and relates the behaviour of a set of units of a system at a given time with the average behaviour of a single unit over time.

In practice, having determined the exponent γ from an empirical cumulative symptom distribution (21), we can predict the average behaviour of a given unit using the symptom life curve (14).
If one can observe several symptoms of evolution of the same system, having determined their γ values, one can choose the most sensitive symptom based on the principle of minimal γ value[4].

For example, in the field of economics we observe for the firms of a given country regularities of type (21): for annual sales, profits, number of employees, capitalization etc. According to our theory, these are different symptoms of the same underlying energy transformation process. For prediction we should select the most sensitive symptom, i.e. the one with the lowest γ value.
Moreover, having determined the minimal γ symptoms for systems of different nature, like the examples shown in this paper, we can try to draw some conclusions as below.

Type of evolving system	γ Pareto	comment
Universe (chem.el.)	0.146 (Fig. 9)	very old
Biological (species)	0.36 (Fig. 6)	old
Urban (cities)	0.94 (Fig. 5)	young
Language (words)	0.99 (Fig. 7)	young
Economic (incomes)	2.39 (Fig. 3)	very young

At hand of the above examples, we have shown the general possibilities of our theoretical approach, but we leave it to the specialists of a given field of science, to do a more in depth analysis.

5 Conclusion

All processes of self-organization or evolution produce symptoms of the Pareto-Zipf type on any observable level of organization (*second law of genesis*).

It was shown here, that the presentation of Pareto-Zipf data in terms of normalized cumulative probability allows to:
i) order data in a common and comparable way. For example see the chemical element distributions of Figure 8, which can be described by a simple law after "ordering" (Figure 9)
ii) to make inferences and draw conclusions from the different values of the exponent γ. Previously the coefficients γ and β in Pareto or Zipf distributions were only mathematical fitting parameters. In our model γ has meaning in terms of energy transformation and life-time
iii) for an individual unit, in the presented model of open systems with limited dissipation capacity, the coefficient γ determines the behaviour over its life-time (see the symptom life curve of Figure 15)

iv) if applied to a set of units (system), the model reveals **Pareto-like behaviour**, not possible to explain by previous statistical considerations (see[12] for example). Recent studies of this problem have shown, that this model gives general results in the form of Frechet distributions[5]. The Pareto distribution is the asymptotic approximation of Frechet only

v) the coefficient γ permits to make inferences from the observation of a set of units concerning their future behaviour and the average life extpectancies of the units constituting the system.

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Appendix: Generalized life symptoms

The presented model "explains" the Pareto-Zipf regularities in terms of a general process of evolution:

1) *Dissipation:*
on a micro-level we observe the dissipation of energy quanta.
Depending on the type of energy transformation, the unit accumulates structural quanta within a limited internal potential. These internally formed structures are *in-formation* in the initial sense of the word.

2) *Development:*
on an intermediate or unit-level, we observe the development of units from "birth" to "death" following a symptom-life curve.

i. By *birth* we understand the beginning of a dissipative energy transformation process within a unit. By *death* we understand the end or break-down of the dissipative energy transformation process within the unit.
ii. *Birth* and *death*, and hence *life*, can be observed for energy transformation processes of a nested hierarchy of energy types: electromagnetic, gravitational, nuclear, inorganic chemical, bio-chemical, economic, "mental" and "symbolic" energy.
iii. *Natural death* occurs, when internally accumulated dissipation energy, its *in-formation* in the form of structure, reaches a limit value.

3) *Evolution:*
on a metasystem-level or macro-level we observe

i) a *global optimization* of energy flows (load optimization) for a set of "running" or "living" units
ii) design or *code modification* for the next *generation* of units (adaptation)
iii) re-design, complete recoding or code-creation for an entire *new technology* (evolution).

4) *Recursiveness:*
In the field of Geometry *fractals* represent a self-similarity of geometrical shape independent of scale.
In the field of processes, like development and evolution, we can speak of *fractal-like processes* i.e. a self-similarity of structuring energy transformation processes independent of scale and type of energy.

Table 2 gives an overview of the fields for which Pareto-Zipf distributions are observed:

Stars and nuclear energy transformation
Nuclides and atoms are "frozen" in-formation of star life-cycles

Planets and inorganic chemical energy transformation
Chemical compounds and bio-molecules are "frozen" in-formation of planet life-cycles

Ecosystems and bio-chemical energy transformation
Species and genera of plants and animals are "frozen" in-formation of ecosystem life-cycles.

Economical systems and money transformation
Goods and money are "frozen" in-formation of economic life-cycles.
Cities are "frozen" in-formation or the infrastructure of economic life cycles

Socio-cultural systems and mental (linguistic) energy transformation
Written words are "frozen" in-formation of cultural life-cycles.

Scientific systems and formal (symbolic) energy transformation
Formal theories are "frozen" in-formation of scientific research life cycles.

It goes beyond the scope of this paper, to think through in detail the application of our model to all the cited fields, but we hope to stimulate the reader to analyze "his special field" in the light of the just presented theory of limited energy dissipation capacity .

Table 2 *Energy transformation systems for which we can observe Pareto-Zipf distributions (generalized "Life-symptoms")*

Evolution and Pareto-Zipf distributions

EVOLUTION	UNIT*	SYSTEM	measured Symptoms	structural *QUANTA***	type of structural energy
cosmic	star planet satellite	galaxy solar system planetary system	light-intensity periodic trajectories	***mass***	gravitational binding
	star meteorite earth crust	universe/galaxy solar system planetary system	spectral intensity	***chem. elements (nucleides)***	nuclear binding (strong & weak interaction)
geological	tectonic region	earth crust	earthquake intensity	***plates, blocs***	tectonic "clustering"
chemical	ocean	earth	chem. element frequency	***anorganic chemical compounds***	anorg. chem. binding
bio-chemical	biotope	biosphere	compound frequency	***organic compounds***	organic chem. binding
biological	family genus species cluster of individuals	ecosystem	class sizes	***DNA, RNA (genes)***	"sequential" and "structural" binding
socio-economic	country city	world country	city-sizes inhabitants	***infrastructure***	"economic" binding
	business firm	world economy economic system economic sector	*firm-sizes* *sales/profits/* *employees* *market value*	***assets***	
	individual	economic system	income		
cultural	text	language	world frequencies	***symbols/sentences***	"grammatical" binding
technical	machine	plant	vibration amplitudes	***infrastructure (damage quanta)***	"structural" binding (micro-fissions)
scientific	author	research area	citation frequencies	***publications***	"formal language" binding

***undergoing birth and death processes**

****irreversibly accumulated information within a unit**